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Information acquisition during a Dutch auction



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The views expressed in this paper are those of the author and do not necessarily reflect the views of the Bank of Finland.

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Abstract

In this paper we consider equilibrium behavior in a Dutch (descending price) auction where the bidders are uninformed of their valuations with probability $1-q$ and can acquire information about their valuation at a positive cost *during the auction*. We assume that the information acquisition activity is covert. We characterize the equilibrium behavior in a setting where bidders are ex ante symmetric and have independent private values. We show that, if the number of bidders is large, the Dutch auction produces more revenue than would a first price auction.

Keywords: auctions, information acquisition

JEL classification numbers: D44, D82, D83

Informaation hankinta hollantilaisen huutokaupan aikana

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Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tässä tutkimuksessa selvitetään tasapainokäyttäytymistä ns. hollantilaisessa eli laskevan hinnan huutokaupassa. Tarjoajat, jotka eivät tiedä huutokaupattavaan hyödykkeeseen kohdistuvaa arvostustaan todennäköisyydellä $1-q$, voivat hankkia informaatiota tavarán arvosta huutokaupan aikana. Informaatio hankitaan muiden tarjoajien näkemättä. Näin ollen tarjoajat eivät tiedä, kuinka moni muista tarjoajista tietää oman arvostuksensa. Tutkimuksessa luonnehditaan tasapainokäyttäytymistä, kun tarjoajat ovat ex-ante-symmetrisiä ja tarjoajien arvostukset ovat jakautuneet riippumattomasti toisistaan. Tarjoajan arvostus objektia kohtaan ei riipu muiden tarjoajien arvostuksista. Päättulos on, että kun tarjoajien määrä on riittävän suuri, hollantilainen huutokauppa tuottaa myyjälle enemmän tuloja kuin ensimmäisen hinnan huutokauppa.

Avainsanat: huutokaupat, informaation hankinta

JEL-luokittelu: D44, D82, D83

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1 Introduction

The theory of auctions usually assumes that the bidders know their valuations for the object to be auctioned. However, there are many instances where this may not be the case: When a venture capitalist is trying to sell a business that it owns it is not immediately clear how much is the company worth for a potential buyer. In addition, if the venture capitalist is unable to sell the company to some set of firms with a given price, he is pushed to lower the price that he asks (or refrain from selling). A lower price may attract the interest of some additional firms. Firms that were not initially interested in the company may want to assess how much the company is worth for them as the price is lowered. Similarly a company that contemplates entering into a takeover battle for one of its rivals must first evaluate how much the rival firm is worth.

Levin and Smith (1994) take up the question of endogenous entry in auctions. In their model the bidders have to incur a positive cost in order to participate into the auction. By paying the participation cost the bidders also learn their valuations for the object. After the bidders have decided about participation the number of participants is made common knowledge. The object is then auctioned to the participating bidders. In a symmetric equilibrium the bidders mix with respect to their decision to participate into the auction. Once the number of participants is clear bidding follows the regular equilibrium behavior in the corresponding auction.

When a static auction is in question this approach is fine, as there is only ‘one round of bidding’. In a dynamic auction, such as a Dutch or an English auction, the ‘decision to participate’ can also be made *during* the auction. That is, if the bidders are allowed to participate into the auction after it has started. The example involving a venture capitalist above fits into this kind of a situation. Another example, that shares the descending nature of prices, is the After-Christmas sales. The sales typically start with a specific discount percentage. The discount percentage is then increased as the sales proceeds.¹ These examples suggest that participation decisions during the auction deserve attention for reasons that are not purely theoretical.

In this paper we study the bidding and information acquisition behavior in the following setting: There are N bidders that can be active in the auction. Each bidder knows his valuation with probability q . Each initially uninformed bidder may become informed by incurring a cost of $c > 0$. We assume that every bidder, informed or not, is allowed to participate into the auction. That is, a bidder may bid for the object even if he is not informed.² We study

¹This year the sales after Christmas in some of Finland’s department stores started with 40 per cent discount. The discount increased up to 70 per cent towards the end of January. The sales for clothing for example, typically involves many items and is not an exact match to the model presented here unless only one item remains in store. However, two features are in common: 1.) the price for the goods in the sale descends 2.) the buyers must incur the cost of inspecting if their size is left in the store.

²The analysis remains the same when c is interpreted as the cost of participating and becoming informed if it is assumed that the seller does not disclose any information about the number of participants. This is because there is no uninformed bidding takes place in the equilibrium that we consider.

the Dutch auction where we assume that each uninformed bidder can decide the price at which he acquires information. If the object is not sold before the ‘information acquisition price’ the bidder becomes informed about his valuation and incurs the cost c . The bidder may then end the auction immediately, or wait for the price to descend further. We also study the first price sealed bid auction in this setting. In the first price auction the information acquisition decisions precede bidding.

We assume that the bidders’ decision to acquire information is covert. Hence each bidder only knows the number of potential competitors. At any given time a bidder does not know how many other bidders have already acquired information or how many other bidders were initially informed.

We consider the case where the bidders have independent private values. We solve for the symmetric equilibrium in the Dutch and in the first price sealed bid auction. In the Dutch auction the uninformed bidders mix with respect to the price at which the information is acquired. The bidding is determined by a pure strategy (conditional on the acquired information). In the first price auction we study the case where the uninformed bidders choose not to acquire information in equilibrium. In the first price auction the information acquisition is discouraged when the number of bidders becomes large. In this case the informed bidders use a pure strategy and the uninformed bidders determine their bids by a mixed strategy. We then compare the revenues that the first price auction and the Dutch auction produce when the number of bidders grows large. We show that in this case the Dutch auction produces larger revenue than the first price auction.

Related literature

Papers that are most related to the issue of information acquisition during the auction are by Bergemann and Välimäki (2005), Compte and Jehiel (2006) and Rezende (2005). Compte and Jehiel (2006) consider information acquisition during an ascending price auction. They allow for information acquisition at any point during the auction in a setting where each bidder has a chance of being informed about his valuation. They show that the ascending price auction can generate higher revenue than the second price sealed bid auction. The setup of this paper coincides with the one analyzed by Compte and Jehiel (2006). Rezende (2005) also studies an ascending price auction, but in his model the bidders have initial estimates about their valuations and they may learn their exact valuation during the auction. He assumes that the bidders’ initial estimates provide statistical information about their true valuation. Additionally, Rezende assumes that the cost of information acquisition is private information and that other bidders’ drop out points are not observed before the auction ends, contrary to the paper by Compte and Jehiel. He characterizes the equilibrium information acquisition strategy and shows, like Compte and Jehiel, that the ascending auction is revenue superior (in some cases) to the second price auction.

Bergemann and Välimäki (2005) survey the literature on information and mechanism design and they emphasize the importance of further work on sequential information acquisition in dynamic auctions. Other related work on information acquisition in static auctions and mechanism design are by

Milgrom (1981), Persico (2000) and Bergemann and Välimäki (2002). Both Milgrom (1981) and Persico (2000) study a situation where the decision to acquire information is made before any bidding takes place. Milgrom (1981) studies the incentives to acquire information in a second price auction while Persico (2000) studies the incentives that the first price and second price auctions provide for information acquisition. Bergemann and Välimäki (2002) study a general mechanism design problem and ask when it is the case that a mechanism provides ex-ante efficient information acquisition incentives and implements the efficient outcome ex-post.

This paper is organized as follows. In section 2 we first introduce the model. We give the reader some flavor of the equilibrium before delving into the proofs of the equilibrium strategies. We then derive equilibrium strategies for the Dutch and the first price auctions. Section 3 works out an approximation for the revenues that one obtains in a Dutch auction and in the first price sealed bid auction. We then show that the Dutch auction produces more revenue than the first price auction when n is large. Section 4 concludes.

2 The model

There are $n \geq 2$ bidders with i.i.d. valuations θ_i , generically denoted by θ . The valuations are distributed on $[0, \bar{\theta}]$ according to an absolutely continuous distribution function $F(\cdot)$, with a density $f(\cdot)$. Each bidder knows his valuation with probability $q > 0$. Hence the number of informed bidders is binomially distributed. The bidders who do not know their valuation may acquire information about their valuation by incurring a cost $c > 0$. We assume that the bidders cannot distinguish between the uninformed and informed bidders. That is, we assume that the information acquisition is covert. We analyze both the Dutch and the first price sealed bid auction. In a Dutch auction the auctioneer begins with a high asking price which is lowered until some participant announces his willingness to buy the object at the current price. This participant wins the auction and pays the current price. In the Dutch auction we assume that the uninformed bidders can acquire information at any price during the auction. In a first price auction the auctioneer asks the bidders to submit sealed bids for the object. The auctioneer collects the bids and declares the bidder with the highest bid as a winner. The winning bidder pays his bid. In the first price auction the information must be acquired prior to the bidding stage. That is, the bidders first make decisions about the information acquisition and then submit sealed bids to the auctioneer. To ease the notation in the paper we denote by $\tilde{G}(x) = (qF(x) + 1 - q)^{n-1}$ and by $G(x) = F(x)^{n-1}$. The corresponding density functions are denoted by $\tilde{g}(x)$ and $g(x)$.

A sketch of the Dutch auction equilibrium

To get a flavor of the equilibrium it is useful to start by considering the information acquisition decision. We argue that the information acquisition must take place over an interval of prices. Consider what happens if the information acquisition were to take place at a specific price. If the price

is ‘low’ the informed bidders have an incentive to bid slightly before the price, since competition intensifies after the price is reached. If the price is ‘high’, the uninformed bidders have an incentive to wait for others to acquire information first and acquire information if the price descends enough. This carries the information that the other bidders’ valuations are not high and that the chances of winning the auction are good. On the other hand if the auction ends soon after the information acquisition price the bidder who decided to wait saves the information acquisition cost. Therefore, in equilibrium, the information acquisition price is decided by mixing over an interval of prices.

Since the information is acquired over an interval, it means that the problem that the informed agents face changes when the price arrives to the information acquisition range. This is because the amount of potential competitors increases. The equilibrium that we derive builds on the existence of a threshold valuation w_* . The bidder with this valuation is indifferent between bidding any price over the mixing interval. When the price is in the mixing interval, a bidder with a valuation higher than w_* wants to buy immediately and a bidder with a valuation lower than w_* wants to wait for the price to descend. Our assumptions guarantee that all bidders are willing to acquire information in the auction. Therefore, in equilibrium, it is common knowledge that all bidders are informed once the price has reached the lower bound of the mixing interval. With these observations as our guide we now proceed to the equilibrium bidding strategies.

The Dutch auction equilibrium strategies

We first make the following assumptions that concern the size of the information acquisition cost c .

Assumption 1. The information acquisition cost $c > 0$ satisfies

$$c = \int_{w_*}^{\bar{\theta}} (x - w_*) f(x) dx \quad (2.1)$$

for some $w_* \in [0, \bar{\theta}]$.

Assumption 2. The information acquisition cost $c > 0$ and the distribution function F satisfy

$$\int_v^{w_*} G(y) dy \geq \int_0^{w_*} G(y) F(y) dy \quad (2.2)$$

Assumptions 1 and 2 are needed for the constructed equilibrium to exist. In practice they imply that the information acquisition cost $c > 0$ should not be too large. One immediate consequence from assumption 2 is that $w_* > v$. It can be readily checked that assumptions 1 and 2 are satisfied with an information acquisition cost $c = 0.01$, $n = 5$ and when the distribution is uniform, exponential or beta.³ Naturally the smaller the cost of information

³For example, the exponential distribution with parameters, $\lambda = 2$ and 5 works and respectively the beta distribution with parameters $(\alpha, \beta) = (1, 2)$ and $(2, 1)$. I think that for a small c and large enough n the assumptions 1 and 2 can be satisfied also for a normal distribution. However, there are some problems involved with recovering the critical value w_* .

acquisition and the larger the amount of bidders the easier it is to fulfill the assumptions. We are now ready for the equilibrium strategies.

Proposition 1. *The following strategies constitute a symmetric Bayesian Nash equilibrium of the Dutch auction.*

- The informed bidders choose the amount they bid according to

$$\tilde{\beta}(\theta) = \begin{cases} \frac{\int_0^\theta x \tilde{g}(x) dx + \mathbf{c}}{\tilde{G}(\theta)}, & \text{if } \theta \geq w_* \\ \frac{\int_0^\theta x g(x) dx}{G(\theta)}, & \text{if } \theta < w_* \end{cases}$$

where $\tilde{G}(x) = (qF(x) + (1-q))^{n-1}$, $G(x) = F(x)^{n-1}$ and $c = \int_0^{w_*} \tilde{G}(x) - G(x) dx$.

- The uninformed bidders choose the price p at which they acquire information from an interval $[\underline{p}, \bar{p}]$, according to the distribution function

$$H(p) = \frac{\left(qF(w_*) + (1-q) \right) \left(\frac{w_* - \bar{p}}{w_* - p} \right)^{\frac{1}{n-1}} - F(w_*)}{(1-q)(1 - F(w_*))}$$

with the property that $H(\underline{p}) = 0$ and $H(\bar{p}) = 1$. After information is acquired the (un)informed bid according to the informed bidders strategy. If the information acquisition price p is such that $\tilde{\beta}(\theta) \geq p$ then the uninformed bidder bids p .

We now prove proposition 1 with the following five lemmata. The lemmas 1 and 2 show that the strategy is optimal for the informed bidders. In lemma 3 we derive the mixing distribution for the uninformed and the end points of the mixing interval. Finally lemmas 4 and 5 show that there are no profitable deviations for the uninformed bidders.

Lemma 1. *The informed bidders with valuations $\theta \geq w_*$ have no profitable deviation.*

Proof. Consider the informed bidder with type $\theta \geq w_*$. His expected payoff from bidding according to $\tilde{\beta}(\cdot)$ is given by

$$\begin{aligned} \mathbb{E}(\tilde{\beta}(\theta)) &= \sum_{k=0}^{n-1} \binom{n-1}{k} q^k (1-q)^{n-1-k} F(\theta)^k (\theta - \tilde{\beta}(\theta)) \\ &= \tilde{G}(\theta) (\theta - \tilde{\beta}(\theta)) \\ &= \int_0^\theta \tilde{G}(x) dx + \mathbf{c} \end{aligned} \tag{2.3}$$

The expected payoff while he is bidding as if his type was z is given by

$$\begin{aligned}
\mathbb{E}(\tilde{\beta}(z)) &= \tilde{G}(z)(\theta - \tilde{\beta}(z)) \\
&= \tilde{G}(z)(\theta - z) + \int_0^z \tilde{G}(x)dx + \mathbf{c}
\end{aligned} \tag{2.4}$$

Subtracting the equation (2.4) from (2.3) we obtain

$$\mathbb{E}(\tilde{\beta}(\theta)) - \mathbb{E}(\tilde{\beta}(z)) = (z - \theta)\tilde{G}(z) - \int_\theta^z \tilde{G}(x)dx \geq 0$$

where the inequality holds irrespective of $z \geq \theta$. ■

Lemma 2. *The informed bidders with valuations $\theta \leq w_*$ have no profitable deviation.*

Proof. Consider the bidding problem for the informed agent, when the price $b \in (\underline{p}, \bar{p})$. The informed agent chooses b to maximize

$$\begin{aligned}
\mathbb{E}(u(b)) &= \sum_{k=0}^{n-1} \binom{n-1}{k} q^k (1-q)^{n-1-k} \left(F(w_*) + (1-F(w_*))H(b) \right)^{n-1-k} F(w_*)^k (\theta - b) \\
&= \left(qF(w_*) + (1-q)(F(w_*) + (1-F(w_*))H(b)) \right)^{n-1} (\theta - b)
\end{aligned}$$

The probability that the informed bidder wins and gets a payoff of $\theta - b$ consists of the following three events. All informed bidders valuations are below w_* , which refers to the term $qF(w_*)$. The uninformed bidders have valuations below w_* , which refers to the term $(1-q)F(w_*)$. Finally the uninformed bidders whose valuations are above w_* have not acquired information prior to b , which refers to the term $(1-q)(1-F(w_*))H(b)$. Substituting for $H(b)$ in the expected utility we obtain after a bit of algebra that

$$\mathbb{E}(u(b)) = \tilde{G}(w_*) \left(\frac{w_* - \bar{p}}{w_* - b} \right) (\theta - b)$$

Now it is immediate that, if $\theta = w_*$ the expected utility is a constant. In addition, the expected utility is increasing in b if $\theta > w_*$ and decreasing in b if $\theta < w_*$. The initially uninformed bidders with $\theta > w_*$ want to bid immediately once they obtain information about their valuation, since their expected utility decreases when the price decreases. Conversely bidders with a valuation $\theta < w_*$ want to wait for the price to descend as their expected utility increases when the price decreases. It is also clear from this analysis that the critical type w_* is unique.⁴ Finally the optimality of $\tilde{\beta}(\cdot)$ when $p < \underline{p}$ follows from the strategic equivalence between the first price auction and a Dutch auction where the valuations are distributed on $[0, w_*]$. ■

Lemma 3. *The uninformed bidders' mixing is determined by $H(\cdot)$, the mixing interval is $[\underline{p}, \bar{p}]$ and c is defined as in proposition 1.⁵*

⁴If $\theta' > w_*$ were the critical type, then the expected utility would decrease as the price increases conversely to the assumption of being a critical type.

⁵(See omitted proofs at the end for some additional derivations.)

Proof. We need that the uninformed bidder is indifferent between acquiring information at any price p on the interval (\underline{p}, \bar{p}) . The expected utility for the uninformed bidder from acquiring information at the price p is

$$W = \int_0^{w_*} (x - \tilde{\beta}(x))G(x)f(x)dx + \left(qF(w_*) + (1-q)(F(w_*) + H(p)(1-F(w_*)))\right)^{n-1} \left((w_* - p)(1-F(w_*))\right)$$

Since $H(\cdot)$ is a distribution function we have that $H(\bar{p}) = 1$. We set $p = \bar{p}$ and get solve for W .

$$W = \int_0^{w_*} (x - \tilde{\beta}(x))G(x)f(x)dx + \tilde{G}(w_*) \left((w_* - \bar{p})(1-F(w_*))\right) \quad (2.5)$$

We substitute W out and solve for $H(p)$ to get

$$H(p) = \frac{qF(w_*) + (1-q)}{(1-q)(1-F(w_*))} \left(\frac{w_* - \bar{p}}{w_* - p}\right)^{\frac{1}{n-1}} - \frac{F(w_*)}{(1-q)(1-F(w_*))}$$

In equilibrium the (initially) informed bidders never bid when the price is in the range (\underline{p}, \bar{p}) . The bidder with the type w_* is indifferent between bidding any price in the interval $[\underline{p}, \bar{p}]$. Therefore $\underline{p} = \beta(w_*) = \frac{\int_0^{w_*} xg(x)dx}{G(w_*)}$ and we can use the fact that $H(\underline{p}) = 0$ to solve for \bar{p} . This yields

$$\bar{p} = \frac{w_* \tilde{G}(w_*) - \int_0^{w_*} G(x)dx}{\tilde{G}(w_*)}$$

By the indifference condition for the type w_* we also need that $\tilde{\beta}(w_*) = \bar{p}$. These equations allows us to pin down the constant of integration c to be

$$c = \int_0^{w_*} \tilde{G}(x) - G(x)dx$$

■

Lemma 4. *The uninformed bidders do not acquire information prior to \bar{p} or wait beyond \underline{p} .*⁶

Proof. Consider first the case where an uninformed agent considers information acquisition prior to \bar{p} . The expected payoff is given by

$$V = \int_0^{w_*} (x - \tilde{\beta}(x))G(x)f(x)dx + \int_{w_*}^{\tilde{\beta}^{-1}(b)} \tilde{G}(x)(x - \tilde{\beta}(x))f(x)dx + \tilde{G}(\tilde{\beta}^{-1}(b)) \left(\int_{\tilde{\beta}^{-1}(b)}^{\bar{\theta}} (x - b)f(x)dx - c \right)$$

Differentiating with respect to the price b we obtain after some manipulations that

$$\frac{d}{db}V = \frac{\tilde{G}(\theta)^2}{\int_0^{\theta} \tilde{G}(x)dx - c} \left(\int_{\theta}^{\bar{\theta}} xf(x)dx - c - \theta(1 - F(\theta)) \right)$$

⁶(See omitted proofs at the end for some additional derivations.)

Since $\frac{\tilde{G}(\theta)^2}{\int_0^{\bar{\theta}} \tilde{G}(x)dx - c} = \frac{\tilde{G}(\theta)}{\theta - \tilde{\beta}(\theta)} \geq 0$ and $\int_0^{\bar{\theta}} xf(x)dx - c - \theta(1 - F(\theta))$ is decreasing in θ we have that $\frac{d}{db}V \leq 0$ for $b \geq \bar{p}$. This is because $\tilde{\beta}^{-1}(b) = \theta > w_*$ and $\int_0^{\bar{\theta}} xf(x)dx - c - \theta'(1 - F(\theta')) = 0$ for $\theta' = w_*$.

Now consider the case where the uninformed is acquiring information at a price $b < \underline{p}$. The expected payoff is given by

$$U = \int_0^{\tilde{\beta}^{-1}(b)} (x - \tilde{\beta}(x))G(x)f(x)dx + \left(\int_{\tilde{\beta}^{-1}(b)}^{\bar{\theta}} (x - b)f(x)dx - c \right) G(\tilde{\beta}^{-1}(b))$$

Differentiating with respect to the price b we obtain after some manipulations that

$$\frac{d}{db}U = \frac{G(\theta)^2}{\int_0^{\bar{\theta}} G(x)dx} \left(\int_0^{\bar{\theta}} xf(x)dx - c - \theta(1 - F(\theta)) \right)$$

Since $\int_0^{\bar{\theta}} xf(x)dx - c - \theta(1 - F(\theta))$ is decreasing in θ we have that $\frac{d}{db}U \geq 0$ for $b < \underline{p}$. This is because $\tilde{\beta}^{-1}(b) = \theta \leq w_*$ and $\int_0^{\bar{\theta}} xf(x)dx - c - \theta'(1 - F(\theta')) = 0$ for $\theta' = w_*$. ■

Lemma 5. *No bidders stay uninformed in equilibrium.*

Proof. We need to show that bidding the best response to $\tilde{\beta}(\theta)$ without acquiring information yields less than bidding according to the equilibrium strategy. Notice that Assumption 2 implies that $v < w_*$. Therefore, the best response to $\tilde{\beta}(\theta)$ for a bidder that remains uninformed is the same as the best response by a bidder whose valuation is exactly v . The expected payoff from bidding $\tilde{\beta}(v)$ is given by

$$(v - \tilde{\beta}_i(v))G(v) = \int_0^v G(y)dy \tag{2.6}$$

The expected equilibrium payoff for an uninformed bidder is derived in equation (2.5). Subtracting equation (2.6) from equation (2.5) and manipulating we get that

$$W - \int_0^v G(y)dy = \int_v^{w_*} G(y)dy - \int_0^{w_*} G(y)F(y)dy,$$

which is non-negative by our Assumption 2. ■

The first price auction equilibrium strategies

We now derive the equilibrium strategies in the first price auction. We focus on the case where the number of informed bidders is so large that information acquisition is an undesirable option for the uninformed bidders.⁷ That is, the cost of information acquisition is larger than the expected payoff for an uninformed bidder who knows that there can be up to $n - 1$ informed bidders

⁷We formalize this assumption below.

in the auction.⁸ This means that the uninformed bidders choose their bids through a mixed strategy.⁹ Note that the problem for the informed bidders is similar to what the informed bidders face in the Dutch auction. When the price is above the upper bound of the mixing interval the informed bidders solve the ‘same’ problem as in (2.3). The only thing that is different is the way that the constant of integration is determined.

The equilibrium has similar features as the Dutch auction equilibrium. In the proof below we use lemma 1 to show that the proposed strategy is an equilibrium for the informed bidders. The differences concern the uninformed bidders’ behavior. This is also where the proof focuses on.

Proposition 2. The following strategies constitute a Bayesian Nash equilibrium of the first price auction, when the information acquisition for the uninformed is too costly.¹⁰ The bidding function for the informed bidders is given by

$$\hat{\beta}_{FPA}(\theta) = \begin{cases} \frac{\int_0^\theta x \tilde{g}(x) dx + \hat{c}}{\tilde{G}(\theta)}, & \text{if } \theta \geq v \\ \frac{\int_0^\theta x g(x) dx}{G(\theta)}, & \text{if } \theta < v, \end{cases}$$

where $\tilde{G}(x) = (qF(v) + (1-q))^{n-1}$, $G(x) = F(x)^{n-1}$ and $\hat{c} = \int_0^v \tilde{G}(x) - q^{n-1}G(x)dx$. The uninformed bidders use a mixed strategy over the interval $[\tilde{p}, \hat{p}]$, where $\tilde{p} = \beta(v)$ and $\hat{p} = \tilde{\beta}(v)_{FPA}$. The mixed strategy distribution function is

$$\hat{H}(\cdot) = \left[\left(\frac{v - \tilde{p}}{v - b} \right)^{\frac{1}{n-1}} - 1 \right] \frac{qF(v)}{1-q}$$

Proof. When the price is above \hat{p} the informed bidders solve the problem in (2.3). Then the difference between the informed bidders’ bid functions in the Dutch and in the first price auction is between constants c and \hat{c} . Since the proof that there are no deviations is identical to the one presented in Lemma 1, we skip it here.

⁸Suppose that there are gains from information acquisition to the uninformed. Assume also that the cost of information acquisition does not accommodate all bidders acquiring information before the auction. Then the uninformed bidders choose whether to acquire information or stay uninformed by using a mixed strategy. This complicates the revenue comparison significantly and I do not pursue this comparison here.

⁹Suppose that there is a price q at which all uninformed bidders bid. This price must be weakly below v to guarantee an expected payoff that is weakly above zero. If $q = v$, then in the case that there is only one uninformed bidder, this bidder can profitably deviate to some $q' < v$. If $q < v$ then any uninformed bidder can profitably deviate by bidding some $q' = q + \epsilon$ for an epsilon small enough. This guarantees virtually the same ex-post payoff as bidding q but with a higher probability, as the bidder avoids the ties that may occur by bidding q .

¹⁰It is enough that the expected payoff from the information acquisition is less than the expected payoff from the uninformed bidding.

If there are uninformed bidders in the auction, then the winning price is always above \tilde{p} . This implies that, in equilibrium, the bidders that have valuations below v never win the auction unless all bidders are informed. Therefore, conditional on a bidder with valuation v having the highest bid, he knows that all other bidders must be informed and have valuations below v . Therefore, the bidders with valuations below v bid according to the regular FPA auction where all bidders valuations are in $[0, v]$.

We now show that the mixing takes place according to the proposed $H(\cdot)$ and that there are no profitable deviations for the informed or uninformed on (\tilde{p}, \hat{p}) . We start by assuming that the mixing by the uninformed takes place on the interval $[\tilde{p}, \hat{p}]$ such that $\tilde{p} = \beta(v)$ and $\hat{p} = \hat{\beta}(v)$ according to the distribution function $\hat{H}(\cdot)$. The uninformed bidders' expected payoff is equal to

$$U = \overbrace{\left(qF(v) + (1-q)\hat{H}(b) \right)^{n-1}}^{\equiv \hat{G}(b)} (v - b)$$

for all $b \in [\tilde{p}, \hat{p}]$. The uninformed bidders' mixed strategy satisfies $\hat{H}(b) = 0$ for $b \leq \tilde{p}$ and $\hat{H}(\hat{p}) = 1$. This information allows us to determine $\hat{H}(\cdot)$ to be

$$\hat{H}(b) = \left[\left(\frac{v - \tilde{p}}{v - b} \right)^{\frac{1}{n-1}} - 1 \right] \frac{qF(v)}{1 - q}$$

Since $\hat{H}(\hat{p}) = 1$ we get that

$$\left(\frac{qF(v) + 1 - q}{qF(v)} \right)^{n-1} = \left(\frac{v - \tilde{p}}{v - \hat{p}} \right)$$

Substituting $\tilde{p} = v - \frac{\int_0^v \left(qF(x) \right)^{n-1} dx}{\left(qF(v) \right)^{n-1}}$, $\hat{p} = v - \frac{\int_0^v \tilde{G}(x) dx - \hat{c}}{\tilde{G}(v)}$ and solving for \hat{c} we find that

$$\hat{c} = \int_0^v \left(qF(x) + 1 - q \right)^{n-1} - \left(qF(x) \right)^{n-1} dx \geq 0$$

Substituting $\hat{H}(\cdot)$ into $\hat{G}(\cdot)$ we get that

$$\hat{G}(b) = \frac{v - \tilde{p}}{v - b} \left(qF(v) \right)^{n-1}$$

Now let's check that none of the informed bidders want to bid $b \in [\tilde{p}, \hat{p}]$. The expected payoff for bidder with type θ from bidding $b \in [\tilde{p}, \hat{p}]$ is

$$\hat{G}(b)(\theta - b) = \frac{v - \tilde{p}}{v - b} \left(qF(v) \right)^{n-1} (\theta - b)$$

If the valuation $\theta = v$, the expected utility is constant for all $b \in [\tilde{p}, \hat{p}]$. If the valuation $\theta > v$ the expected utility increases with b implying that the bidder

wants to bid more than $\hat{p} = \hat{\beta}(v)$. If the valuation $\theta < v$ the expected utility decreases with b implying that the bidder wants to bid less than $\tilde{p} = \beta(v)$.

Since the uninformed bidders do not acquire information their ‘valuation’ is essentially v . Therefore, they do not want to bid above \hat{p} , since this is the optimal bid for the bidder of type v . Similarly they do not want to bid below \tilde{p} since this is the optimal bid for the bidder of type v when all other bidders are informed and have valuations below v . ■

We now formalize our assumption that the uninformed bidders do not acquire information in the FPA. The expected payoff for an uninformed bidder from information acquisition in the FPA is

$$\begin{aligned} \mathbf{R} &= \int_0^{\bar{\theta}} \Pr(\text{win})(x - \hat{\beta}_{FPA}(x))f(x)dx - c \\ &= \int_0^v \int_0^x \left(qF(y)\right)^{n-1} dy f(x)dx \\ &\quad + \int_v^{\bar{\theta}} \int_0^x \left((qF(y) + 1 - q)^{n-1} dy - \hat{c}\right) f(x)dx - c \end{aligned} \tag{2.7}$$

Now by the monotone convergence theorem we have that as $n \rightarrow \infty$ the expected revenue tends to $-\hat{c}(1 - F(v)) - c \leq 0$.

Lemma 6. *For all $c > 0$ and $q > 0$ there exist $m \in \mathbb{N}$ such that if $n \geq m$ then $R < 0$.*

Discussion

In the Dutch auction the uninformed bidders may postpone their information acquisition decision. This allows them to measure the level of competition prior to acquiring information. As the uninformed observe that the price descends the information acquisition becomes more attractive. A lower price implies that the competitors’ valuations are drawn from an interval with a lower upper bound and hence the competitors’ valuations are also smaller in expectation. Lower price also implies that the probability that the uninformed bidder has the highest valuation increases, which is good news for the uninformed. Note that this is in contrast with what is observed by Rezende (2005) in the context of an ascending auction. In his model the bidders do not observe the number of remaining bidders either. It is bad news for the uninformed to observe a price increase in the ascending auction, since it only conveys the information that it is less likely that his valuation is the largest among all bidders. In the ascending price auction, where the number of remaining bidders is not observed, no information about the intensity of competition is available to the (uninformed) bidders.

In the first price auction it is not possible to defer information acquisition. In fact, information acquisition quickly becomes unattractive, when the number of bidders increases. The main reason why the bidding functions differ in the two auctions is that the competition intensifies sooner in the Dutch auction than it does in the first price auction.¹¹ In the Dutch auction the bidders know that the competition intensifies as the uninformed bidders start

¹¹Note that Assumption 2 implies that $v < w_*$. We can show that $\hat{p} \leq \bar{p}$ for large n .

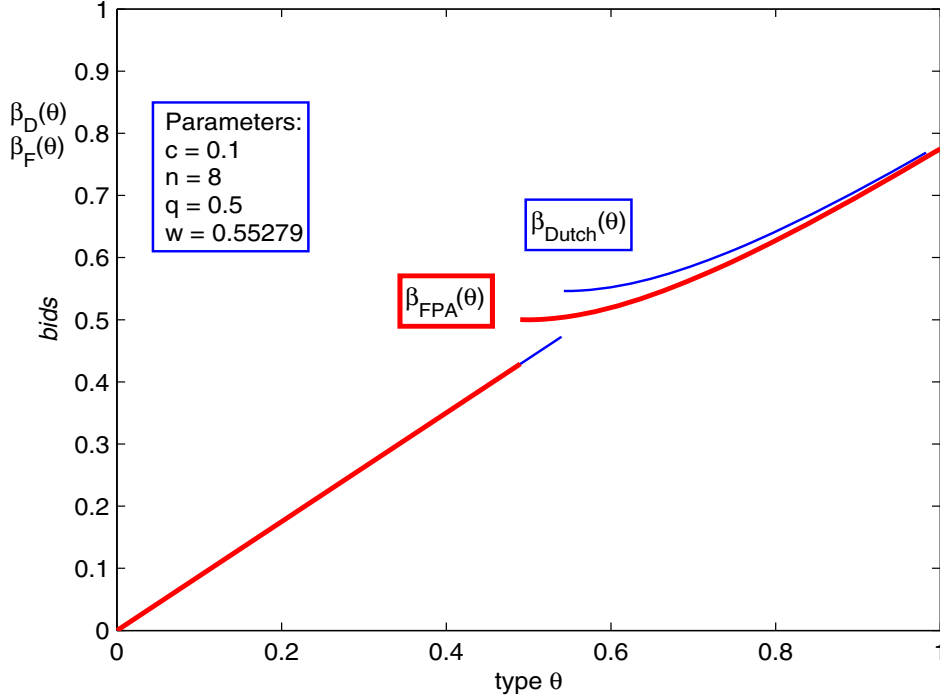


Figure 1: Bid functions for the informed in the FPA and Dutch Auction

to acquire information.¹² In the first price sealed bid auction the uninformed bidders start bidding later than in the Dutch auction. Therefore, for a fixed number of informed bidders, the competition is less intensive in the first price auction.

We highlight this feature with the example in the figure below. Here the valuations are uniformly distributed and we graph the bidding functions for the informed bidders. The uninformed bidders' mixed strategies 'fill the gaps'. In the mixing takes place on the interval where the informed bidders' bid function jumps. Therefore, the distribution of bids has no jumps in it. It should be noted that the first price auction ends with a winning bid that is weakly above \tilde{p} in all cases but the one where all bidders are informed and have valuations below v .

Notice that the first price auction bid function goes above the Dutch auction bid function for a small range of values. However, the Dutch auction bid function stays above the first price auction bid function once they have crossed.¹³ For a given number of uninformed bidders the first price auction bids are in the range $[\tilde{p}, \hat{p}]$, while the Dutch auction bids are in the range $[0, \bar{p}]$. When the number of uninformed bidders increases the expected bid by an uninformed bidder tends towards \bar{p} in the Dutch auction while in the first

¹²Note also that in a regular setting with independent private values, the bidders learn nothing about their opponents during the auction. Here, the bidders learn that all of their opponents are informed, if price descends below \underline{p} .

¹³If $n = 2$ in this example, the Dutch auction bid function never crosses the first price auction bid function. However, with $n = 3$ the crossing occurs.

price auction it tends towards \hat{p} . In this sense the uninformed bidders bid ‘more aggressively’ in the Dutch auction.

On an intuitive level this implies that the revenue from the Dutch auction is larger than from the first price auction when the number of bidders is large enough. This is because with a large number of bidders the probability mass assigned to the events where the Dutch auction bids are above the first price auction bids converges to unity. At the same time the mass that is assigned to the events where the first price auction bids are above the Dutch auction bids becomes very small. We now address this issue formally.

3 The revenue

We begin by calculating the revenues to the seller from the Dutch and the first price sealed bid auction. We calculate a lower bound of the Dutch auction revenue and an upper bound of the FPA revenue. We then show that the revenue approximation for the Dutch auction is larger than the revenue approximation for the FPA.

The Dutch auction revenue

Let’s first calculate the revenue when we know the number of informed bidders.¹⁴ The expected revenue from the Dutch auction with k informed bidders is¹⁵

$$\mathbb{E}[\mathbb{R}_{Dutch}(k)] = k m_i + (n - k) m_u \quad (3.1)$$

where

$$m_i = \int_0^{w_*} \int_0^x y g(y) dy f(x) dx + \int_{w_*}^{\bar{\theta}} \frac{F(x)^{k-1}}{\tilde{G}(x)} \left(\int_0^x y \tilde{g}(y) dy + \mathbf{c} \right) f(x) dx$$

and

$$\begin{aligned} m_u &= \int_0^{w_*} \int_0^x y g(y) dy f(x) dx + (1 - F(w_*)) F(w_*)^k \int_{\underline{b}}^{\bar{b}} \left(F(w_*) \right. \\ &\quad \left. + H(b)(1 - F(w_*)) \right)^{n-k-1} b h(b) db \\ &\geq \int_0^{w_*} \int_0^x y g(y) dy f(x) dx + \frac{F(w_*)^k}{n - k} (1 - F(w_*)^{n-k}) \underline{b} \end{aligned}$$

The interpretation of m_i is straight forward. Either all bidders valuations are below w_* or one of the informed bidders with a valuation $\theta \geq w_*$ wins the auction. Consider then the terms in m_u . Again either all bidders valuations are below w_* or conditional on all informed bidders valuations being below w_*

¹⁴Here the bidders don’t know the number of informed bidders, but given the equilibrium strategies the revenue can be calculated in the case when the number of informed bidders is fixed.

¹⁵(See omitted proofs at the end for some additional derivations (Lemma 7.)

one of the uninformed bidders discovers that his valuation is above w_* and bids at the price that he decided about the information acquisition.

Collecting the terms from above we have that

$$\begin{aligned}
\mathbb{E}[\mathbb{R}_{Dutch}(k)] &= k m_i + (n - k) m_u \\
&\geq \overbrace{n \int_0^{w_*} \int_0^x y g(y) dy f(x) dx}^{D_1} + \overbrace{k \int_{w_*}^{\bar{\theta}} \frac{F(x)^{k-1}}{\tilde{G}(x)} \int_0^x y \tilde{g}(y) dy f(x) dx}^{D_2} \\
&\quad + \overbrace{k \int_{w_*}^{\bar{\theta}} \frac{F(x)^{k-1}}{\tilde{G}(x)} f(x) dx}^{D_3} + \overbrace{k \int_0^{w_*} f(x) F(x)^{k-1} (1 - F(w_*)^{n-k}) dx}^{D_4} \\
&\equiv \mathbb{E}[\mathbb{R}_{aDutch}(k)].
\end{aligned} \tag{3.2}$$

The expected revenue from the Dutch auction is given by

$$\sum_{k=0}^n \binom{n}{k} q^k (1 - q)^{n-k} \mathbb{E}[\mathbb{R}_{Dutch}(k)]$$

The first price auction revenue

We now calculate the revenue from the first price auction. We assume that there are enough potentially informed bidders so that information acquisition is an undesirable option. We know from the analysis of the bidding behavior in the first price auction that the uninformed bidders bids are below v on an interval $[\tilde{p}, \hat{p}]$. Since we only want to show that the Dutch auction provides more revenue than the first price auction we compare the Dutch auction revenue to an upper bound of the first price auction revenue. The upper bound of the first price auction revenue is obtained by calculating the revenue in a first price auction where the seller has a reserve valuation equal to v .¹⁶ That is, the seller is always guaranteed a minimum of v from selling the object. This revenue is clearly higher than the revenue from the first price auction, since the uninformed bidders always bid below v in equilibrium. This results in less aggressive bidding when compared to an auction where the reserve price is set to v . I.e. the informed bidders bid less aggressively in the equilibrium of the first price auction.

Let $\hat{\beta}_{aFPA}(\cdot)$ denote the bidding function in the first price auction with a reservation value equal to v .¹⁷ To see that the informed bidders behave more aggressively in the first price auction where the auctioneer is assumed to have a reservation valuation of v one just needs to show that $\beta_{aFPA}(\cdot) \geq \hat{\beta}_{FPA}(\cdot)$.¹⁸

¹⁶Note that since the uninformed bidders play a mixed strategy the first price auction typically ends with a winning bid that is no less than $\tilde{p} > 0$. Only in the case that all bidders are informed, may the winning price be below \tilde{p} .

¹⁷It is straightforward to show that the equilibrium bidding strategy in the first price auction with a reserve price equal to v is given by $\beta_{aFPA}(\theta) = \int_v^{\theta} \frac{y \tilde{g}(y)}{\tilde{G}(\theta)} dy + \frac{\tilde{G}(v)}{\tilde{G}(\theta)} v$.

¹⁸We leave this to the reader. (See lemma 8 in the omitted proofs at the end.)

We now consider the revenue to the seller whose reserve valuation is v . The revenue to the auctioneer from the first price auction, when there are k informed bidders, is given by¹⁹

$$\begin{aligned}
\mathbb{E}[\mathbb{R}_{aFPA}(k)] &= \overbrace{k \int_v^{w_*} F(x)^{k-1} \int_v^x \frac{y\tilde{g}(y)}{\tilde{G}(x)} dy f(x) dx}^{F_1} \\
&\quad + \overbrace{k \int_{w_*}^{\bar{\theta}} F(x)^{k-1} \int_v^x \frac{y\tilde{g}(y)}{\tilde{G}(x)} dy f(x) dx}^{F_2} \\
&\quad + \overbrace{k \int_v^{\bar{\theta}} \frac{F(x)^{k-1}}{\tilde{G}(x)} f(x) dx \tilde{G}(v) v}^{F_3} + \overbrace{F(v)^k v}^{F_4}
\end{aligned} \tag{3.3}$$

and the expected revenue from this FPA is given by

$$\sum_{k=0}^n \binom{n}{k} q^k (1-q)^{n-k} \mathbb{E}[\mathbb{R}_{aFPA}(k)]$$

We show that the lower bound of the Dutch auction revenue is larger than the upper bound of the FPA revenue. We show that this is true for a generic k and so this proves that the revenue from the Dutch auction is superior to the revenue of the first price auction.²⁰

The Dutch auction produces a superior revenue to the first price auction, if $n \in N$ is large enough.

Proposition 3. *It is sufficient to show that for all $k \leq n$*

$$\mathbb{E}[\mathbb{R}_{aDutch}(k)] - \mathbb{E}[\mathbb{R}_{aFPA}(k)] \geq 0$$

We prove this proposition by comparing the terms in equations (3.2) and (3.3) and by showing that either the terms are positive or that the negative terms tend to zero as n grows. The proposition is established by showing that there remain some strictly positive terms for each k that do not depend on n .

Proof. We omit the straightforward proofs that $D_2 - F_2 \geq 0$ for all $k \leq n$ and that $\lim_{n \rightarrow \infty} D_3 - F_3 \rightarrow 0$ for all $k \leq n$.²¹ We now show that the difference between the terms D_1, D_4 and F_1, F_4 in equations (3.2) and (3.3) satisfies

$$\lim_{n \rightarrow \infty} D_1 + D_4 - F_1 - F_4 \rightarrow C(k) > 0 \text{ for all } k < n, \text{ where } C(k)$$

$$= F(v)^k (w_* - v)$$

and

$$\lim_{n \rightarrow \infty} D_1 + D_4 - F_1 - F_4 \rightarrow 0 \text{ for } k = n$$

¹⁹(See omitted proofs at the end for some additional derivations (Lemma 7.)

²⁰We deal with $k = n$ separately.

²¹(See lemmas 9 and 10 in the omitted proofs.)

Let $k < n$ be fixed. Then we have that

$$\begin{aligned}
D_1 + D_4 - F_1 - F_4 &= (n-1) \int_0^{w_*} yg(y) dy F(w_*) - n \int_0^{w_*} yg(y) F(y) dy \\
&+ k \int_0^v f(x) F(x)^{k-1} dx \int_0^{w_*} \frac{yg(y)}{G(w_*)} dy \\
&+ k \int_v^{w_*} F(x)^{k-1} f(x) dx \left(\int_0^v \frac{yg(y)}{G(w_*)} dy \right. \\
&\quad \left. + \int_v^{w_*} \frac{yg(y)}{G(w_*)} dy \right) \\
&- k \int_v^{w_*} F(x)^{k-1} \int_v^x \frac{y\tilde{g}(y)}{\tilde{G}(x)} dy f(x) dx - F(v)^k v \\
&\geq (n-1) \int_0^{w_*} yg(y) dy F(w_*) - n \int_0^{w_*} yg(y) F(y) dy \\
&+ F(v)^k \int_0^{w_*} \frac{yg(y)}{G(w_*)} dy \\
&+ \left(F(w_*)^k - F(v)^k \right) \left(\int_0^v \frac{yg(y)}{G(w_*)} dy + \int_v^{w_*} \frac{yg(y)}{G(w_*)} dy \right) \\
&- \left(F(w_*)^k - F(v)^k \right) \int_v^{w_*} \frac{y\tilde{g}(y)}{\tilde{G}(w_*)} dy - F(v)^k v \\
&= \overbrace{\int_0^{w_*} yg(y) \left((n-1)F(w_*) - nF(y) \right) dy}^{T_1} \\
&+ \overbrace{F(v)^k \left(w_* - v - \int_0^{w_*} \frac{G(y)}{G(w_*)} dy \right)}^{T_2} \\
&+ \overbrace{\left(F(w_*)^k - F(v)^k \right) \left(\int_0^v \frac{yg(y)}{G(w_*)} dy + \int_v^{w_*} \frac{yg(y)}{G(w_*)} - \frac{y\tilde{g}(y)}{\tilde{G}(w_*)} dy \right)}^{T_3}
\end{aligned}$$

Where the inequality follows from the fact that $\frac{\int_0^x y\tilde{g}(y)dy}{\tilde{G}(x)}$ is increasing in x and $x \leq w_*$. We show that as n increases the terms in the last equation converge to zero or stay positive. Let's consider the first term T_1 , which is not positive:

$$\begin{aligned}
&\int_0^{w_*} xg(x) \left((n-1)F(w_*) - nF(x) \right) dx \\
&= (n-1) \int_0^{w_*} G(x) \left(F(x) - F(w_*) \right) dx \\
&\leq 0,
\end{aligned}$$

because $F(x) - F(w_*) \leq 0$ for all $x \in [0, w_*]$. Now we have that

$$(n-1)G(w_*) \int_0^{w_*} \left(F(x) - F(w_*) \right) dx \leq (n-1) \int_0^{w_*} G(x) \left(F(x) - F(w_*) \right) dx.$$

Since $G(w_*)$ decreases to zero exponentially while $n - 1$ grows at a constant rate we have that

$$\lim_{n \rightarrow \infty} (n - 1)G(w_*) \int_0^{w_*} (F(x) - F(w_*))dx \rightarrow 0$$

and therefore

$$\lim_{n \rightarrow \infty} (n - 1) \int_0^{w_*} G(x)(F(x) - F(w_*))dx \rightarrow 0$$

Then consider the second term T_2 as n grows.²²

$$\lim_{n \rightarrow \infty} w_* - v - \int_0^{w_*} \frac{G(y)}{G(w_*)} dy = w_* - v - \int_0^{w_*} \lim_{n \rightarrow \infty} \frac{G(y)}{G(w_*)} dy = w_* - v > 0$$

Here $\frac{G(y)}{G(w_*)} = \left(\frac{F(y)}{F(w_*)} \right)^{n-1} \geq \left(\frac{F(y)}{F(w_*)} \right)^n \geq 0$, and we use the monotone convergence theorem while taking the limit. Now the constant $C(k)$ is given by

$$C(k) = F(v)^k (w_* - v)$$

Finally consider the last term T_3 and focus on²³

$$\int_v^{w_*} \frac{yg(y)}{G(w_*)} - \frac{y\tilde{g}(y)}{\tilde{G}(w_*)} dy = v \left(\frac{\tilde{G}(v)}{\tilde{G}(w_*)} - \frac{G(v)}{G(w_*)} \right) + \int_v^{w_*} \frac{\tilde{G}(y)}{\tilde{G}(w_*)} - \frac{G(y)}{G(w_*)} dy \quad (3.4)$$

Now

$$\frac{\tilde{G}(y)}{\tilde{G}(w_*)} = \left(\frac{qF(y) + 1 - q}{qF(w_*) + 1 - q} \right)^{n-1} < 1$$

and

$$\frac{G(y)}{G(w_*)} = \left(\frac{F(y)}{F(w_*)} \right)^{n-1} < 1, \text{ for } y < w_*$$

We have that

$$\left(\frac{qF(y) + 1 - q}{qF(w_*) + 1 - q} \right)^{n-1} \geq \left(\frac{F(y)}{F(w_*)} \right)^{n-1} \iff F(w_*) \geq F(y)$$

which is true for all $y \leq w_*$. Hence

$$\int_v^{w_*} \frac{yg(y)}{G(w_*)} - \frac{y\tilde{g}(y)}{\tilde{G}(w_*)} dy \geq 0$$

for all $n, k \in \mathbb{N}$. It is clear that

$$\lim_{n \rightarrow \infty} \frac{\tilde{G}(y)}{\tilde{G}(w_*)} - \frac{G(y)}{G(w_*)} \rightarrow 0$$

²²We drop the multiplier $F(v)^k$ here for convenience.

²³We disregard the first term since $\int_0^v \frac{yg(y)}{G(w_*)} dy \geq 0$ for all $n \in \mathbb{N}$ and $\lim_{n \rightarrow \infty} \int_0^v \frac{yg(y)}{G(w_*)} dy \rightarrow 0$.

In addition, the multiplier term $F(w_*)^k - F(v)^k$ is positive and disregarded here as well.

for all $y \in [v, w_*]$. Then using the monotone convergence theorem, it is immediate from equation (3.4) that

$$\lim_{n \rightarrow \infty} \int_v^{w_*} \frac{yg(y)}{G(w_*)} - \frac{y\tilde{g}(y)}{\tilde{G}(w_*)} dy \rightarrow 0$$

Therefore we can always find $n \in N$ such that the negative terms are less than the positive ones.

Finally let $k = n$. Above we used the fact that $k < n$ when considering the term T_2 . In this case we have that $C(n) = F(v)^n(w_* - v)$ which also converges to zero as n grows to infinity. The term T_3 also has a multiplier that depends on k . The result that the term converges to zero does not change when we have $k = n$. ■

4 Conclusion

In this paper we've examined the bidding behavior in first price and Dutch auctions with independent private values. Some bidders may be uninformed about their valuations and acquire information during the Dutch auction. We solve for equilibrium in both auctions in this setting and show that the Dutch auction produces more revenue to the seller than the first price auction, when the number of bidders is large.

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Appendix

Omitted proofs

Below we refer to the bid of the informed bidder with $\tilde{\beta}$. The bid by the (initially) uninformed bidder is referred to with b_{uni} .

Lemma 3. *We derive the expression for W in the proof of lemma 3.*

The uninformed bidder is indifferent between acquiring information at any price p on the interval (\underline{p}, \bar{p}) . The expected utility for the uninformed bidder from acquiring information at the price p is

$$W = \sum_{k=0}^{n-1} \binom{n-1}{k} q^k (1-q)^{n-1-k} \times \left\{ \begin{aligned} & \mathbb{E} \left[(\theta - \tilde{\beta}(\theta)) \Pr(\theta_{-i} < \theta \mid \theta_{-i} < w_*)^k \Pr(\theta_{-i} < \theta \mid b_{uni} < p)^{n-1-k} \mid \theta < w_* \right] \\ & \Pr(\theta < w_*) \\ & + \mathbb{E} [\theta - p \mid \theta \geq w_*] \Pr(\theta \geq w_*) - c \end{aligned} \right\} \times \Pr(\theta_{-i} < w_*)^k \Pr(b_{uni} < p)^{n-1-k}$$

where $\Pr(b_{uni} < p) = (1 - H(p))F(w_*) + H(p)$. Using Assumption 1 we get that this can be written as

$$\begin{aligned} W &= \sum_{k=0}^{n-1} \binom{n-1}{k} q^k (1-q)^{n-1-k} \int_0^{w_*} (x - \tilde{\beta}(x)) G(x) f(x) dx \\ &+ \sum_{k=0}^{n-1} \binom{n-1}{k} q^k (1-q)^{n-1-k} \left((w_* - p)(1 - F(w_*)) \right) \\ &F(w_*)^k \left((1 - H(p))F(w_*) + H(p) \right)^{n-1-k} \\ &= \int_0^{w_*} (x - \tilde{\beta}(x)) G(x) f(x) dx \\ &+ \left(qF(w_*) + (1-q)(F(w_*) + H(p)(1 - F(w_*))) \right)^{n-1} \left((w_* - p)(1 - F(w_*)) \right) \end{aligned}$$

Lemma 4. *We derive the expressions for V and U in lemma 4.*

The case where an uninformed agent considers information acquisition prior

to \bar{p} . The expected payoff is given by

$$\begin{aligned}
V &= \sum_{k=0}^{n-1} \binom{n-1}{k} q^k (1-q)^{n-1-k} \times \\
&\quad \left\{ \begin{aligned} &\mathbb{E} \left[(\theta - \tilde{\beta}(\theta)) \Pr(\theta_{-i} < \theta \mid \theta_{-i} < \tilde{\beta}^{-1}(b))^k \right. \\ &\quad \times \Pr(\theta_{-i} < \theta \mid b_{uni} < \tilde{\beta}^{-1}(b))^{n-1-k} \mid \theta < w_* \Big] \Pr(\theta < w_*) \\ &+ \mathbb{E} \left[(\theta - \tilde{\beta}(\theta)) \Pr(\theta_{-i} < \theta \mid \theta_{-i} < \tilde{\beta}^{-1}(b))^k \mid \theta \in [w_*, \tilde{\beta}^{-1}(b)) \right] \\ &\quad \times \Pr(\theta \in [w_*, \tilde{\beta}^{-1}(b))) \\ &+ \mathbb{E}[\theta - b \mid \theta \geq \tilde{\beta}^{-1}(b)] \Pr(\theta \geq \tilde{\beta}^{-1}(b)) - c \end{aligned} \right\} \\
&\quad \times \Pr(\theta_{-i} < \tilde{\beta}^{-1}(b))^k \Pr(b_{uni} < \tilde{\beta}^{-1}(b))^{n-1-k} \\
&= \int_0^{w_*} (x - \tilde{\beta}(x)) G(x) f(x) dx + \int_{w_*}^{\tilde{\beta}^{-1}(b)} \tilde{G}(x) (x - \tilde{\beta}(x)) f(x) dx \\
&\quad + \tilde{G}(\tilde{\beta}^{-1}(b)) \left(\int_{\tilde{\beta}^{-1}(b)}^{\bar{\theta}} (x - b) f(x) dx - c \right)
\end{aligned}$$

The case where the uninformed is acquiring information at a price $b < \underline{p}$. The expected utility from this is given by

$$\begin{aligned}
U &= \sum_{k=0}^{n-1} \binom{n-1}{k} q^k (1-q)^{n-1-k} \times \\
&\quad \left\{ \begin{aligned} &\mathbb{E} \left[(\theta - \tilde{\beta}(\theta)) \Pr(\theta_{-i} < \theta \mid \theta_{-i} < \tilde{\beta}^{-1}(b))^k \right. \\ &\quad \times \Pr(\theta_{-i} < \theta \mid b_{uni} < b)^{n-1-k} \mid \theta < \tilde{\beta}^{-1}(b) \Big] \Pr(\theta < \tilde{\beta}^{-1}(b)) \\ &+ \mathbb{E}[\theta - b \mid \theta \geq \tilde{\beta}^{-1}(b)] \Pr(\theta \geq \tilde{\beta}^{-1}(b)) - c \end{aligned} \right\} \\
&\quad \times \Pr(\theta_{-i} < \tilde{\beta}^{-1}(b))^{n-1} \\
&= \int_0^{\tilde{\beta}^{-1}(b)} (x - \tilde{\beta}(x)) G(x) f(x) dx + \left(\int_{\tilde{\beta}^{-1}(b)}^{\bar{\theta}} (x - b) f(x) dx - c \right) G(\tilde{\beta}^{-1}(b))
\end{aligned}$$

Lemma 7. *Derivations for the expressions in the revenue section.*

Dutch. In the text above we assumed that there are k informed bidders in the auction that we are studying. A few definitions are in order before going forward with the derivations. Here Y_n^1 refers to the first order statistic in a sample of size n . When considering the uninformed bidder, we abuse notation by referring with the term $\Pr(\tilde{\beta}_{-i}, b_{uni} < b) = F(w_*)^k \left(F(w_*) + H(b)(1 - F(w_*)) \right)^{n-k-1}$ to the probability that all other bids are below $b > w_*$. The informed bidders bid below $b \geq w_*$ with probability $F(w_*)^k$ and all the

other uninformed bidders bid below b with probability $\left(F(w_*) + H(b)(1 - F(w_*))\right)^{n-k-1}$.

$$\begin{aligned}
m_i &= \int_0^{w_*} \Pr(Y_{n-1}^1 < x) \tilde{\beta}(x) f(x) dx + \int_{w_*}^{\bar{\theta}} \Pr(Y_{k-1}^1 < x) \tilde{\beta}(x) f(x) dx \\
&= \int_0^{w_*} \int_0^x y g(y) dy f(x) dx + \int_{w_*}^{\bar{\theta}} \frac{F(x)^{k-1}}{\tilde{G}(x)} \left(\int_0^x y \tilde{g}(y) dy + \mathbf{c} \right) f(x) dx \\
m_u &= \int_0^{w_*} \Pr(Y_{n-1}^1 < x) \tilde{\beta}(x) f(x) dx + \int_{\underline{b}}^{\bar{b}} \Pr(\tilde{\beta}_{-i}, b_{uni} < b) b h(b) db \Pr(\theta \geq w_*) \\
&= \int_0^{w_*} \int_0^x y g(y) dy f(x) dx + (1 - F(w_*)) F(w_*)^k \\
&\quad \times \int_{\underline{b}}^{\bar{b}} \left(F(w_*) + H(b)(1 - F(w_*)) \right)^{n-k-1} b h(b) db
\end{aligned}$$

FPA

$$\begin{aligned}
\mathbb{E}[\mathbb{R}_{aFPA}(k)] &= \int_v^{\bar{\theta}} \Pr(Y_{k-1}^1 < x) \beta_{aFPA}(x) f(x) dx + \Pr(Y_k^1 < x) v \\
&= k \int_v^{\bar{\theta}} F(x)^{k-1} \left(\int_v^x \frac{y \tilde{g}(y)}{\tilde{G}(x)} dy + \frac{\tilde{G}(v)}{\tilde{G}(x)} v \right) f(x) dx + F(v)^k v \\
&= k \int_v^{w_*} F(x)^{k-1} \int_v^x \frac{y \tilde{g}(y)}{\tilde{G}(x)} dy f(x) dx + k \int_{w_*}^{\bar{\theta}} F(x)^{k-1} \int_v^x \frac{y \tilde{g}(y)}{\tilde{G}(x)} dy f(x) dx \\
&\quad + k \int_v^{\bar{\theta}} \frac{F(x)^{k-1}}{\tilde{G}(x)} f(x) dx \tilde{G}(v) v + F(v)^k v
\end{aligned}$$

Lemma 8. $\beta_{aFPA}(\cdot) \geq \hat{\beta}_{FPA}(\cdot)$.

Proof.

$$\begin{aligned}
\beta_{aFPA}(\theta) - \hat{\beta}_{FPA}(\theta) &= \int_v^{\theta} \frac{y \tilde{g}(y)}{\tilde{G}(\theta)} dy + \frac{\tilde{G}(v)}{\tilde{G}(\theta)} v - \int_0^{\theta} \frac{x \tilde{g}(x)}{\tilde{G}(\theta)} dx - \frac{\hat{\mathbf{c}}}{\tilde{G}(\theta)} \\
&= - \int_0^v \frac{y \tilde{g}(y)}{\tilde{G}(\theta)} dy + \frac{\tilde{G}(v)}{\tilde{G}(\theta)} v - \frac{\hat{\mathbf{c}}}{\tilde{G}(\theta)} \\
&= - \frac{v \tilde{G}(v)}{\tilde{G}(\theta)} + \int_0^v \frac{\tilde{G}(y)}{\tilde{G}(\theta)} dy + \frac{\tilde{G}(v)}{\tilde{G}(\theta)} v - \int_0^v \frac{\tilde{G}(x) - \left(qF(x)\right)^{n-1}}{\tilde{G}(\theta)} dx \\
&= \frac{\int_0^v \left(qF(x)\right)^{n-1} dx}{\tilde{G}(\theta)} \geq 0
\end{aligned}$$

■

Lemma 9. The difference between the terms D_2 and F_2 in equations (3.2) and (3.3) satisfies

$$D_2 - F_2 \geq 0 \text{ for all } k \leq n \text{ and all } n \in \mathbb{N}$$

Proof. Taking the difference between the terms D_2 and F_2 we have that

$$\begin{aligned} & k \left(\int_{w_*}^{\bar{\theta}} \frac{F(x)^{k-1}}{\tilde{G}(x)} \int_0^x y \tilde{g}(y) dy f(x) dx - \int_{w_*}^{\bar{\theta}} F(x)^{k-1} \int_v^x \frac{y \tilde{g}(y)}{\tilde{G}(x)} dy f(x) dx \right) \\ &= k \int_{w_*}^{\bar{\theta}} \frac{F(x)^{k-1}}{\tilde{G}(x)} \int_v^x y \tilde{g}(y) dy f(x) dx \geq 0 \end{aligned}$$

for all $n, k \in \mathbb{N}$. ■

Lemma 10. The difference between the terms D_3 and F_3 in equations (3.2) and (3.3) satisfies

$$\lim_{n \rightarrow \infty} D_3 - F_3 \rightarrow 0 \text{ for all } k \leq n$$

Proof. We disregard the term $D_3 \geq 0$ here, although it can be shown that it converges to zero. We concentrate on the term F_3 in equation (3.3) and show that it converges to zero as n increases. First let $k < n$ be fixed. Then

$$F_3 = k \int_v^{\bar{\theta}} \frac{\tilde{G}(v)}{\tilde{G}(x)} F(x)^{k-1} f(x) dx v$$

where $\frac{\tilde{G}(v)}{\tilde{G}(x)} = \left(\frac{qF(v)+1-q}{qF(x)+1-q} \right)^{n-1} < 1$ for all $x \in (v, w_*]$. Since $\left(\frac{qF(v)+1-q}{qF(x)+1-q} \right)^{n-1} \geq \left(\frac{qF(v)+1-q}{qF(x)+1-q} \right)^n \geq 0$ and $\lim_{n \rightarrow \infty} \frac{\tilde{G}(v)}{\tilde{G}(x)} \rightarrow 0$ for all $x \in (v, w_*]$ we have by the monotone convergence theorem that

$$\lim_{n \rightarrow \infty} k \int_v^{\bar{\theta}} \frac{\tilde{G}(v)}{\tilde{G}(x)} F(x)^{k-1} f(x) dx v = k \int_v^{\bar{\theta}} \lim_{n \rightarrow \infty} \frac{\tilde{G}(v)}{\tilde{G}(x)} F(x)^{k-1} f(x) dx v \rightarrow 0$$

Now if $k = n$ then we have that

$$\lim_{n \rightarrow \infty} n \int_v^{\bar{\theta}} \frac{\tilde{G}(v)}{\tilde{G}(x)} F(x)^{n-1} f(x) dx v = \lim_{n \rightarrow \infty} n \tilde{G}(v) \int_v^{\bar{\theta}} \frac{F(x)^{n-1}}{\tilde{G}(x)} f(x) dx v \rightarrow 0$$

since

$$\lim_{n \rightarrow \infty} n \tilde{G}(v) \rightarrow 0$$

as n increases at a fixed rate while $\tilde{G}(v)$ decreases at an exponential rate and because $F(x)^{n-1} = G(x) \leq \tilde{G}(x)$ for all $x \in [v, \bar{\theta}]$ we have that

$$\lim_{n \rightarrow \infty} \int_v^{\bar{\theta}} \frac{G(x)}{\tilde{G}(x)} f(x) dx v = \int_v^{\bar{\theta}} \lim_{n \rightarrow \infty} \frac{G(x)}{\tilde{G}(x)} f(x) dx v \rightarrow 0$$

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